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Seventy presents were announced as having been received since the last meeting, including, amongst others :

Annales de l'Observatoire de Meudon—Atlas de Photographies Solaires, presented by M. Janssen ; Observatoire d'Alger, Catalogue Photographique du Ciel, 3 fascicules ; and Introduction by Ch. Trépied, presented by the Observatory ; G. E. Hale and F. Ellerman (&c.), The Rumford Spectroheliograph of the Yerkes Observatory, and Spectra of Stars of Secchi's fourth type, presented by Professor Hale ; E. B. Frost and W. S. Adams, Radial Velocities of Stars of the *Orion* type, presented by Professor Frost ; S. W. Burnham, Measures of Double Stars, presented by the Author ; Tokyo Observatory, Annales, tomes 2, 3 (with report of Solar Eclipse of 1901), presented by the Observatory.

On the Degree of Accuracy of the new Lunar Theory and on the Final Values of the Mean Motions of the Perigee and Node.

By Ernest W. Brown, Sc.D., F.R.S.

In the *Monthly Notices* for 1903 May I gave a general statement of the degree of precision existing between the calculated values of the mean motions of the perigee and node and their observed values. It was added that the final verifications of the higher terms due to solar action were not complete, but an upper limit was given for possible errors. Since that date the calculations for these higher terms have been tested and found to be correct, so that the portions due to solar action are completely finished, and the portion arising from each characteristic is calculated to $0''\cdot01$ in the annual motions.

In order to make the comparison complete to this degree of accuracy it was found necessary to undertake an examination into the numerical values of the constants used and into the effects produced by sources other than the direct solar action. This inquiry revealed several differences which had a perceptible effect. It was first necessary to develop a general method of dealing with the effects of planetary and other perturbations on these mean motions which would permit of their being found easily and accurately. Two methods for the solution of this problem were obtained ; they will be published elsewhere,* the numerical results only being given here. It was found necessary to make several changes in the values of these non-solar perturbations as collected in my papers in the *Monthly Notices* for 1897 March and June : four of these caused alterations of about half a second each in the annual motions. First, in multiplying the ratios of the motions to n—ratios which were found correctly from theory— by n (the mean motion), it was found that a

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* *Trans. Amer. Math. Soc.* July 1904.

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slightly inaccurate value of n had been used. Second, Hansen's value for the planetary action on the perigee was found to be wrong by a quarter of its whole amount. Third, Hill's values for the effects produced by the figure of the earth correspond to an ellipticity rather greater than would seem to be admissible. Fourth, Hansen's and Newcomb's values, deduced from observation, correspond to the epoch 1800, while the values of the constants used in my theory, especially that of the solar eccentricity, correspond to the epoch 1850. After these changes had been made it was found that the agreement between theory and observation was not impaired; in fact, the differences are still within the limits of error arising from doubts as to the correct values to be assigned to certain constants. As far as the lunar theory is concerned, every theoretical result set down here has been calculated or tested afresh by one, or in some cases two methods; the only exception is in a case where little doubt can be entertained and where in any case the effect is very small—namely, the perturbations of the solar radius vector, and these have been taken from Newcomb's *Tables of the Sun.*

There is only one constant whose observed value is so far doubtful as to affect the results by so much as 0″·1—the ellipticity of the earth. For this constant there seem to be really two competing values, one about 1/293 and the other about 1/297. As it appears difficult to make a choice between them I give two calculated results corresponding to two values of the ellipticity, the final determinations are as follows:

Annual Mean Motions, Epoch 1850·0.

	Perigee.			Node.		
		″	″		″	″
Calculated (α)	+146	435·27	±0·10,	−69	679·37	∓0·05;
Calculated (β)	+146	435·11	±0·10,	−69	679·22	∓0·05;
Observed	+146	435·23	... ,	−69	679·45	... ;
C−O (α)	...	+0·04	... ,	...	+0·08	... ;
C−O (β)	...	−0·12	... ,	...	+0·23	... ;

(α) corresponds to an ellipticity 1/292·9;
(β) „ „ „ 1/296·3;

the quantities ±0″·10, ∓0″·05 being extreme possible errors due to neglect of terms whose orders are higher than those calculated and to the doubt as to the correct value of the mass of *Venus.* The direct and indirect effects due to *Mercury* are nearly equal and opposite, so that the uncertainty as to the mass of this planet does not affect the results. No possible errors due to the use of a limited number of places of decimals have been set down, since nearly every number has been computed to at least one more place than that actually given; the errors from this cause are probably less than 0″·05.

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The portions which various parts of the complete disturbing function produce will now be considered in detail. So far as I know, every purely gravitational cause which can give a sensible effect has been considered; the list in No. 7 below will, in any case, permit of any changes or additions to be easily made.

The references are collected at the end of the paper and are referred to by the letters (*a*), (*b*), ...

The adopted values of the constants are for the epoch 1850·0:

Moon.	Sun.
$n = 17\,325\,594''$,	$n' = 1\,295\,977''\cdot 4$;
$e = \cdot 054\,900\,3$,	$e' = \cdot 016\,772$;
$\gamma = \cdot 044\,887\,2$,	$\gamma' = 0$;
Parallax $= 3422''\cdot 7$,	Parallax $= 8''\cdot 80$;
$\mathrm{E}/\mathrm{M} = 81\cdot 5$,	$m'/(\mathrm{E}+\mathrm{M}) = 328\,243$;

where n, n' are the annual mean motions; E, M, m' the masses of the Earth, Moon, Sun; e, the lunar eccentricity corresponding to Newcomb's coefficient $22659''\cdot 58$ for the principal elliptic term (*a*); e', the solar eccentricity (*b*); γ, the sine of half the inclination corresponding to Newcomb's coefficient $18461''\cdot 48$ for the principal term in latitude (*a*); the parallaxes are those generally accepted at the present time. No probable change in any of these constants will materially alter the values for the mean motions.

1. *The solar portions.*—These are obtained by using the disturbing function (*a*)

$$(\mathrm{E}+\mathrm{M}+m')\left[\frac{1}{\sqrt{(x-x')^2+(y-y')^2+z^2}}-\frac{xx'+yy'}{r'^3}\right]$$

$$=\frac{n'^2a'^3}{r'^3}\left\{r^2\left(\tfrac{3}{2}\mathrm{S}^2-\tfrac{1}{2}\right)+\frac{r^3}{a'}\left(\tfrac{5}{2}\mathrm{S}^3-\tfrac{3}{2}\mathrm{S}\right)+\frac{r^4}{a'^2}\left(\tfrac{35}{8}\mathrm{S}^4-\tfrac{15}{4}\mathrm{S}^2+\tfrac{3}{8}\right)\right\}$$

where the solar coordinates x', y', z', r' refer to the elliptic motion of the Sun about the Earth with the correction

$$\frac{\mathrm{E}-\mathrm{M}}{\mathrm{E}+\mathrm{M}}\cdot\frac{a}{a'}=a \text{ for } \frac{a}{a'}$$

in the final results—a correction which accounts for the greater part of the motion of E about the centre of mass of E and M. Here S is the cosine of the angle between the radii to the Sun and Moon.

The portions calculated are those with characteristics 1, e^2, γ^2, e'^2, $(a/a')^2$, e^4, $e^2\gamma^2$, γ^4, $e^2e'^2$, $\gamma^2e'^2$. A consideration of the numerical values of other characteristics shows that the sum of the terms which arise from them will be less than $0''\cdot 04$ in the case of the perigee and less than $0''\cdot 02$ in the case of the node.

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The terms of characteristic unity form respectively 71/70, 174/175 of the complete values.

In an earlier paper (*d*) I have given results (extensions of Adams' theorems) which directly connected the parts in the motion of the perigee having characteristics $e^2\gamma^2$, γ^4, $\gamma^2 e'^2$ with those in the motion of the node having characteristics e^4, $e^2\gamma^2$, $e^2 e'^2$ respectively. The former set are determined in the theory with the terms in x, y (longitude and parallax), and the latter with the terms in z (latitude), which are of the fifth order with respect to their characteristics. The two sets are calculated in totally different ways, the former with the "homogeneous" equations and the latter with the non-homogeneous equation. There were thus three separate tests, and in all cases the results agreed within two units of the last place of decimals used. These searching tests, involving nearly all the results of the third order, a large proportion of those of the fourth order, and the greater part of the forms of computation for the terms of the fifth order, would seem to warrant a feeling of security in the general accuracy of the whole theory, so far as it has gone.

But these three tests, though they are exact and searching, are on the whole less valuable than an indirect test for general accuracy which arises automatically in every set of terms with a given characteristic. Each such set in the x, y coordinates and every other set in the z coordinate has one (long-period) or two (monthly) terms whose coefficients possess small divisors, and the process of division by these divisors is practically the last step in the determination of the coefficients. Now, the possibility of error in the *details* of numerical calculation has been practically eliminated; the chief danger arises from the possible omission of a whole set of terms, or the use of a wrong set. But it is precisely the latter forms of error which the test guards against, for such mistakes would almost certainly cause the coefficients to be many times greater than their actual values, and a rough comparison with the results of Delaunay would show the existence of error at once. In fact, the numbers before division are composed of quantities which are large in comparison with their algebraical sum. Thus, the chief cause of the enormous extent of the computations furnishes the most valuable test of their accuracy.

2. *Correction due to using* E + M + *m′ instead of m′ as the factor of the disturbing function.*—The additional sensible portion is (*c*)

$$-\frac{E+M}{E+M+m'}\, n'^2 r^2 \left(\tfrac{3}{2}S^2 - \tfrac{1}{2}\right).$$

The terms of characteristic unity, due to this function, have been calculated by the method of variation of arbitraries as well as by the method of the theory, and the results agreed in giving $-0''\cdot71$ for the perigee and $+0''\cdot20$ for the node. Multiplying these by 70/71, 175/174 respectively, which accounts sufficiently

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for the terms in higher characteristics, we obtain −0″·70, +0″·20, agreeing with the estimate which I gave in an earlier note (*e*). No possible alteration in the value attached to the ratio $(E+M)/(E+M+m')$ will change these by so much as 0″·01.

3. *Correction for the remaining effect of the mass of the Moon.*—The addition to the disturbing function (*c*) is a term

$$\frac{EM}{(E+M)^2} n'^2 \frac{r^4}{a'^2} \left(\tfrac{35}{8} S^4 - \tfrac{15}{4} S^2 + \tfrac{3}{8}\right).$$

It gives +0″·02 for the perigee, −0″·01 for the node. The indirect action produced by a corresponding term in the Sun's disturbing function gives −0″·01 and 0″·00 in the respective cases.

4. *Direct planetary action.*—This has been calculated by the method of the theory (for its principal part) and also completely by the variation of arbitrary constants. Using Hansen's values for the masses of the planets, and his expansions of the disturbing functions, only in so far as they depend on the positions of the Earth and planets, it appears that Hansen's result for the perigee is much too small (*f*). Adopting modern values for the masses and Newcomb's expansion for *Venus* (*h*), which has the greatest effect, I find +2″·66 for the perigee and −1″·42 for the node. Hansen (*f*) gave +2″·10 and −1″·50. My results include squares of the eccentricities and inclinations of the Moon and planets, and all powers of *m*. The use of these more accurate values for the parts depending on the planets would subtract about 0″·1 from Hansen's result for the perigee, so that it is 0″·7 in error. The node is practically unaffected. I have not been able to trace the source of this error in Hansen very far; it depends, however, on the terms factored by K in Art. 172 of the *Darlegung*, the value of K being correctly found with the adopted values of the masses.

The following table gives the values of the masses adopted here (*b*) with the corresponding motions and the extreme amounts by which the latter can be in error owing to the uncertainty in the mass of *Mercury*, which is doubtful to 50 per cent. of the given mass, and in the mass of *Venus*, which is doubtful to 5 per cent.:

Direct Action of Planets.

	Recip. of Mass.	Perigee.	Node.
		″	″
Mercury	6,000,000	+0·049 ± ·025	−0·016 ∓ ·008
Venus	408,000	+1·966 ± ·098	− ·819 ∓ ·041
Mars	3,093,000	+ ·034	− ·025
Jupiter	1,047·35	+ ·582	− ·536
Saturn	3 501· 6	+ ·026	− ·025
Sum	...	+2·657 ± 123	−1·421 ∓ ·049

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The extreme errors, when taken in connection with those produced by the indirect action, are diminished to one-half of these values, as appears in No. 6 below.

5. We have put, in the solar disturbing function after the correction of No. 2 has been made,

$$\mathrm{E}+\mathrm{M}'+m' = n'^2 a'^3, \quad r' = a' + \text{elliptic terms.}$$

The latter value of a' requires a correction, the constant part of which will be denoted by $\delta a'$, on account of the perturbations produced in the Sun's motion by the planets. The addition to the disturbing function for the Moon is

$$-3\frac{\delta a'}{a'} n'^2 r^2(\tfrac{3}{2}\mathrm{S}^2-\tfrac{1}{2}).$$

It is therefore similar to that of No. 2, and the corrections to the perigee and node are respectively

$$-\frac{3\delta a'}{a'}\cdot\frac{0''\cdot70}{328243}, \quad +\frac{3\delta a'}{a'}\cdot\frac{0''\cdot20}{328243}.$$

The value of $\delta a'/a'$ for *Mercury* has been obtained from Newcomb (*i*) and for the other planets and for the adopted value of the gravitation constant from his tables of the Sun (*b*). The corrections, with the extreme errors due to uncertainty in the adopted masses of *Mercury* and *Venus*, are given in the following table :

Indirect Planetary Action.

	$\log_{10}(1+\delta a'/a')$.	Perigee.	Node.
Mercury ...	+ ·000 000 028	−0″·041 ∓ ·020	+0″·012 ± ·006
Venus ...	+ 634	− ·934 ∓ ·047	+ ·267 ± ·013
Mars	− 12	+ ·018	− ·005
Jupiter ...	− 512	+ ·754	− ·215
Saturn ...	− 24	+ ·036	− ·010
Grav. const. ...	+ 13	− ·019	+ ·005
Sum	+ ·000 000 127	−0″·186 ∓ ·067	+0″·054 ± ·019

The action of *Venus* produces a periodic inequality with a coefficient of about 15″. When squares of the action are included, this inequality will give rise to small changes in the mean motions of the perigee and node. These I have examined and found to be absolutely insensible.

6. *Total planetary action.*—It will be noticed that the direct and indirect actions of *Mercury* are opposite in effect and nearly equal, so that any uncertainty as to its mass does not affect the results by so much as 0″·01. The indirect action of *Venus* is about half the direct action with the opposite sign.

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Hence, combining the results in Nos. 4, 5, we can subtract the two sets of extreme errors and obtain for the

Total Planetary Action.

Perigee.	Node.
$+2''\cdot48 \pm 0''\cdot06$	$-1''\cdot37 \mp 0''\cdot03$,

the extreme errors arising entirely from doubt as to the mass of *Venus.*

7. *Figure of the Earth effects.*—The part of the disturbing function due to the figure of the earth, which alone can give rise to sensible terms in the mean motions of the perigee and node, is

$$\lambda\left(\frac{1}{3}\frac{a^3}{r^3}-\frac{z^2\,a^3}{r^5}\right).$$

Hill (k) has made an extended literal investigation by Delaunay's method of the terms arising from this function, but it is open to the objection that the results for the perigee and node are only carried as far as the orders λm^3, λe^2, $\lambda\gamma^2$. It appears from the expression for the perigee, which begins with a term of order λm^0, that slow convergence along powers of m might make a difference of as much as $0''\cdot3$ from Hill's result. I therefore made a new and different computation, using the numerical value of m and carrying the approximations so far as to include all sensible terms of the forms

$$\lambda f(m),\ \lambda e^2 f(m),\ \lambda\gamma^2 f(m),\ \lambda e'^2 f(m).$$

The terms of the form $\lambda f(m)$ were also computed by the method of my theory. It appeared that the apparent slow convergence had very little effect. Hill's results for the perigee and node were $+6''\cdot82$, $-6''\cdot41$ respectively, while mine gave $+6''\cdot86$, $-6''\cdot42$, using Hill's value for λ. The only possible source of error is therefore in the value to be substituted for λ.

The usual method, and probably the best one for the purposes of the lunar theory, is to determine λ by means of the coefficient of the principal term, due to the figure of the Earth, in latitude. This, according to Hill, has a coefficient $-8''\cdot726$, and, excluding the doubt as to λ, it is probably accurate within $0''\cdot003$. Hansen (f) finds $-8''\cdot382$ for the observational value of the coefficient. There is a term due to the motion of the ecliptic with the same argument, and with a coefficient, according to Hill's theory (l), $-0''\cdot226$ (according to Hansen $-0''\cdot240$); hence the part of the observed coefficient due to the figure of the Earth is $-8''\cdot156$. We must therefore multiply the numbers $6''\cdot86$, $-6''\cdot42$, obtained with Hill's value of λ, by $8\cdot156 \div 8\cdot726$ to reduce them to the value deduced from Hansen's observed coefficient in latitude. They respectively become $+6''\cdot41$, $-6''\cdot00$.

Faye (m), however, gives $-8''\cdot59$ for the observed coefficient in latitude, so that the part due to the Earth's action is $-8''\cdot36$. The corresponding motions are $+6''\cdot57$, $-6''\cdot15$. Helmert (m) has a still different value.

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The coefficient λ, according to the theories of the figure of the Earth, is approximately equal to $k\,(e_1 - \frac{1}{2}\phi)$, where k is a constant determinable with all the accuracy required here, ϕ is the ratio of centrifugal force to gravity at the equator and is equal to ·003468, and e_1 is the ellipticity of the Earth's surface.* As is well known, this equation is obtained on certain suppositions concerning the constitution and distribution of the Earth's mass. Hill (k) finds λ from a collection of pendulum observations, and deduces $1/e_1 = 286{\cdot}7$. Helmert (m) has deduced $1/e_1 = 298{\cdot}3 \pm 1{\cdot}0$, also from pendulum observations. Clarke (m), from geodetical measurements, obtains $1/e_1 = 293{\cdot}5 \pm 1{\cdot}0$. The precessional constant used by Newcomb (b) is connected by the theory of the figure of the Earth with the value 297·3. As the value of the ellipticity deduced from Hill is greater than most of the other determinations, it seems worth while to give a list of the principal determinations, together with the corresponding motions of the perigee and node :

	Author.	$\frac{1}{e_1}$	Annual Motions.	
			Perigee.	Node.
	Hill, pendulum obs.	286·7	$+6''{\cdot}86$	$-6''{\cdot}42$
(α)	Faye, coef. in lat.	292·9	$+6{\cdot}57$	$-6{\cdot}15$
	Clarke, geodesy	293·5	...	...
(β)	Hansen, coef. in lat.	296·3	$+6{\cdot}41$	$-6{\cdot}00$
	Precessional const.	297·3	...	...
	Helmert, Moon pert.	297·8	...	...
	Helmert, pendulum obs.	298·3	$+6{\cdot}33$	$-5{\cdot}94$

Hill's theoretical coefficient in latitude is compared with Faye's observed coefficient in order to find e_1 in (α) of the table, and Hansen's observed coefficient is similarly treated, so that Hansen's theoretical coefficient does not enter into this discussion.

According to the evidence Hill's value is too large. Faye's value puts the differences between the observed and theoretical motions of the perigee and node within the limits of errors arising from other known sources. Hansen's value is close to that obtained by Helmert, and also to that deduced from the precessional constant. I shall therefore in the final results, awaiting a new discussion of the observed value of the coefficient in latitude, give the two values (α), (β), these being practically the values round which a choice seems probable; they also make the theory consistent as far as the adopted values

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* The ellipticity is defined by $e_1 = (a-c)/a$, where a, c are the equatorial and polar axes of the Earth. The complete relation between λ, e_1 is given by Professor Darwin in the *Monthly Notices* for 1899 December. I am much indebted to Professor Darwin for assistance in the discussion of this paragraph, and to Professor Helmert for communicating a calculation of the values of ϕ, k, from the relation just mentioned, and of the latest result, $298{\cdot}3 \pm 1{\cdot}0$ for $1/e_1$ (*Berl. Sitzungsber.*, 1901, pp. 328–336).

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of the constants are concerned. A change of 1 per cent. in the ellipticity causes changes of 2 per cent. in the corresponding perturbations of the Moon's motion.

8. *Other perturbations.*—Hansen (*f*) considers the figure of the Moon as responsible for the differences he finds between the observed and theoretical motions—namely, +1″·50, −2″·56. As will be seen, no such differences exist, and in any case modern measurements of the Moon seem to put the large values Hansen assigned for the ellipticity of the Moon entirely out of the question.* In fact, until the question of the ellipticity of the Earth is settled it is unnecessary to invoke any other sources of perturbation beyond those which can be definitely calculated. In particular the law of gravitation at the distance of the Moon appears to be exact within the limits of accuracy at present attainable.

9. *The final results.*—Gathering together the results in the foregoing sections, we obtain the following :

Annual Mean Motions, Epoch 1850·0.

		Perigee.		Node.	
Solar Action	char^c. 1	+148 524″·92		−69 287″·90	
	,, e^2	− 519·31		− 616·09	
	,, γ^2	− 1 739·85		+ 260·59	
	,, e'^2	+ 156·27		− 25·46	
	,, a^2	+ 2·24		− 1·11	
	,, e^4	+ ·04		+ ·07	
	,, $e^2\gamma^2$	+ 6·72		− 1·70	
	,, γ^4	− 1·51		+ ·05	
	,, $e^2e'^2$	− ·99		− ·57	
	,, $\gamma^2e'^2$	− 1·61		+ ·08	
	,, rem^g.	·00 ± ·04		·00 ∓ ·02	
Terms in No. 2		− ·70		+ ·20	
Terms in No. 3		+ ·01		− ·01	
Plan. direct		+ 2·66	± ·06	− 1·42	∓ ·03
Plan. indirect		− ·19		+ ·05	
Fig. of earth (α)		+ 6·57		− 6·15	
Fig. of earth (β)		+ 6·41		− 6·00	
Calcul. sum (α)		+146 435·27 ± ·10		−69 679·37 ∓ ·05	
Calcul. sum (β)		+146 435·11 ± ·10		−69 679·22 ∓ ·05	
Observed		+146 435·23		−69 679·45	
C−O (α)		+ 0·04 ± ·10		+ 0·08 ∓ ·05	
C−O (β)		− 0·12 ± ·10		+ 0·23 ∓ ·05	

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* It is to be noticed that if we add 1/12 of these differences to the result (β) in No. 9, the differences C−O (β) practically disappear. But both sets are too doubtful to build any argument on their possible existence.

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Of the calculated results (α), (β) in the above table, the former is obtained by using Faye's value of the observed coefficient in latitude to determine the constant on which the terms due to the figure of the Earth depend, the latter by using Hansen's observed coefficient. It is probably true that Hansen's coefficient refers to his special axes, but the difference between his and the ecliptic system will not materially alter it. The errors $\pm\cdot10$, $\mp\cdot05$ are the extreme errors previously explained.

Nothing has been said concerning the probable errors of the observed values of the mean motions. In view of Newcomb's (*a*) confirmation of Hansen's (*f*) results (a change of $0''\cdot1$ in the motion of the node alone being made), it would seem that there is little doubt of their accuracy as far as Hansen's theory with the known corrections is concerned. But of late years the examination of planetary inequalities has revealed several whose periods are very close to that of the principal elliptic term. Three of them have coefficients approaching $1''$ in magnitude, according to Radau (*n*), and it is not impossible that a systematic search may reveal others whose inclusion in the theory may cause slight alterations in the observed values of the mean motions of the perigee and node. The values adopted above are those of Newcomb (*a*).

10. *Bibliography.*—The references above are to the following papers :

(*a*) S. Newcomb, "Investigation of Corrections to Hansen's Tables of the Moon," *Transit of Venus Papers*, pt. 3, Washington, 1876.

(*b*) S. Newcomb, "Tables of the Sun," *Washington Astr. Papers*, vol. vi. pt. 3, 1895.

(*c*) E. W. Brown, "Theory of the Motion of the Moon," pt. 1. *Mem. R.A.S.* vol. liii. 1897.

(*d*) E. W. Brown, "On certain Properties of the Mean Motions...," *Proc. Lond. Math. Soc.* vol. xxviii. 1896.

(*e*) E. W. Brown, "On the Mean Motions of the Lunar Perigee and Node," *M.N.R.A.S.* vol. lvii. 1897. Note on the same, *ib.*

(*f*) P. A. Hansen, "Darlegung...," pt. 1, *Sächs. Abh.* vol. vi. 1862.

(*h*) S. Newcomb, "Action of the Planets on the Moon," *Washington Astr. Papers*, vol. v. pt. 3, 1894.

(*i*) S. Newcomb, "Periodic Perturbations . . . of the Four Inner Planets . . ." *Washington Astr. Papers*, vol. iii. pt. 5, 1891.

(*k*) G. W. Hill, "Inequalities . . . produced by the Figure of the Earth," *Washington Astr. Papers*, vol. iii. pt. 2, 1891.

(*l*) G. W. Hill, "On the Lunar Inequalities produced by the Motion of the Ecliptic," *Annals of Mathematics*, vol. i. 1884.

(*m*) These references are extracted from F. Tisserand, *Méc. Cél.* vol. ii. chap. 21, and W. Harkness, "Solar Parallax and Related Constants," *Washington Observations for* 1885, app. 3.

(*n*) R. Radau, "Recherches concernant les Inégalités plané-

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taires du mouvement de la Lune," *Par. Obs. Ann.* (*Mém.*) vol. xxi. 1892.

The work of Harkness (*m*) contains a collection of all the constants of astronomy and a full bibliography.

Haverford College: 1904 *January* 4.

The Parallactic Inequality and the Solar Parallax.

By Ernest W. Brown, Sc.D., F.R.S.

Of the principal solar terms in the Moon's motion the parallactic inequality is the only one whose coefficient can be determined correctly within $0''\cdot1$ when characteristics of order higher than the third are omitted. Like all the terms whose coefficients contain a/a' as a factor, the convergence of its coefficient along powers of m is slow, and consequently its value deduced from Delaunay's literal expression, even when allowance is made for higher powers of m, is not certain within half a second. The transformation of my theory to polar coordinates up to and inclusive of the terms whose characteristics are of order 3 was completed some time ago (*Monthly Notices*, 1899 December). As the coefficient of the parallactic inequality can be found, with a given set of constants, correctly to $\frac{1}{5000}$ of the whole from these terms it may perhaps be useful to give the result.

The following table gives the values of the various portions of the coefficient obtained by myself and Delaunay. It is expressed exactly in the form and with the constants of Delaunay, who uses a solar parallax of $8''\cdot75$. The column headed l.p. *m.* gives the value of the last power of *m* calculated by Delaunay. The result has not been multiplied by (E−M)/(E+M), the factor necessary to obtain the final value.

Coefficient of Parallactic Inequality.

Characteristic.	Brown.	Delaunay.	l.p. *m*.
$a/a' = \alpha$	$-128''\cdot069$	$-127''\cdot621$	$-0''\cdot381$
$e^2\alpha$	$-\ 2\cdot485$	$-\ 2\cdot378$	$-\ \cdot119$
$e'^2\alpha$	$-\ \cdot022$	$-\ \cdot024$	$+\ \cdot001$
$p^2\alpha$	$+\ 3\cdot105$	$+\ 2\cdot772$	$+\ \cdot354$
α^3	$-\ \cdot001$	$\cdot000$	$-\ \cdot0004$
Higher	...	$+\ \cdot009$	$+\ \cdot009$
Sum	$-127\cdot472$	$-127\cdot242$	...

My result is thus $-127''\cdot47$ + higher terms. The latter, judging from Delaunay's calculation and the experience gained as to the